An evaluation of the accuracy of DEM-derived altitude and slope values

Technical Report

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Figure 2. Location of the study area.
Plate I. Perspective views of the study area.
Plates 2 and 3. Photographs of the study area.
An evaluation of the accuracy of DEM-derived altitude and slope values

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An evaluation of the accuracy of DEM-derived altitude and slope values

Abstract  This project was set up to investigate the level of accuracy which can be expected for slope and altitude values derived from low cost Digital Elevation Models (DEMs). Light gridded DLMs were generated from digitised contour maps at a range of scales (from 1:10,000 to 1:200,000) and using a range of contour intervals (25 m, 50 m and 100 m). A Control DEM was then produced using large scale aerial photographs (1:28,000) which were registered for auto extraction of z values using Helava software and accuracy tested using 91 differentially measured GPS ground control points. The DEM showed a vertical RMSE of 4.26 m which is well within the accuracy standards for a level one DEM as stipulated by the USGS. The altitude and slope readings derived from each of the eight test models were then compared to the values derived from the Control to assess the relationship between the cost of production of a DEM and the accuracy of the results. The relationship between cell size and slope correlation was also examined. Several recommendations are made regarding optimal production methods for a DLM based on application needs.
1 Introduction

A digital elevation model (DEM) is a three-dimensional computerised model of the earth's surface used to store topographic attributes in digital form. These models have been developed within the field of Geographical Information Systems (GIS) and are a valuable source of data for agricultural research. The information they provide can be used as input to a wide range of projects such as soil erosion modelling (Vertessy et al. 1990), crop suitability (Bradley et al. 1994), drainage basin monitoring and flood control (Rosenthal et al. 1995), hydrological run-off modelling (MacMillan et al. 1994), land classification (Robison et al. 1992, Dikau 1989), viewshed analysis (Lee 1991, Smart et al. 1991) and pollution dispersion modelling (Woodrow 1993). They are also being used in the field of Remote Sensing to aid geometric and radiometric correction of satellite images (Conese et al. 1993).

It is widely acknowledged that the products of a GIS will reflect and in some cases augment, any errors present in the source data (Goodchild and Gopal 1989). Likewise, the accuracy of a DEM and its products is dependent on the quality of the altitude data from which the surface has been generated.

To generate and store DEMs, many data sources can be used ranging from analytical and softcopy photogrammetry of stereo satellite imaging and aerial photographs (Day and Müller 1988, Welch and Papacharalampous 1992, Toutin and Beaudoin 1995) to the digitisation of topographic maps (Eklundh and Martensson 1995) and ground surveys (McLaren and Kennie 1989). The range of data sources for deriving DEM surfaces is diversifying rapidly, and the sources need to be investigated and assessed for their reliability. Accuracy assessments of DEMs
and their products appear in scientific journals (Skidmore 1989, Adkins and Merry 1994, Bolstad and Stowe 1994, Brown and Barra 1994, Gao 1997). The United States Geological Survey (USGS) published a set of accuracy specifications for their own DEM products in 1990 (USGS 1990). To date no international standards exist to which a DEM should conform to lend legitimacy to its resulting data. Many of the accuracy assessments that have been performed are specific to a single data source product in a particular type of landscape.

Findings from current research in this field are non-transferable or inapplicable to the data and complex landscapes encountered in tropical hillside areas. However, the Centro Internacional de Agricultura Tropical (CIAT) needed information to validate DEMs in their agricultural models for the hillsides of tropical America. A further problem for CIAT research programs was the lack of information regarding the costs associated with producing a DEM and their relationship with the level of accuracy that can be expected. The data sources for producing DEMs that are believed to be more accurate are also understood to be more expensive (Rhund 1992), although little investigation has been made of this relationship and of quantifying the level of performance of data in the low-cost categories.

Specific objectives of the project were to

- Determine the level of accuracy of slope and altitude values derived from DEMs that have been produced using different scales of cartographic data.
- Examine the relationship between increased accuracy of a DEM and corresponding increases in the cost of production.
- Examine the relationship between cell resolution and the correlation between "true" and derived slope values
- Examine high error occurrences document where they are, develop reasons for their occurrence, and work towards reducing them

Finally it was hoped to increase the user's awareness of the level of accuracy they can expect of DLMs being used

2 Methods

The evaluation was divided into four phases (see 2.1 to 2.4) each of which deals with a particular stage of the research, development, and assessment of nine DEMs. Eight of these models were produced using a range of topographic maps as source data, and the ninth using highly accurate, large-scale stereo photography. The ninth was considered to represent "true" altitude and was developed to act as a control. This model was quality controlled using many Global Positioning Systems (GPS) ground control points (GCPs) collected in the field

2.1 Study area

The study area was selected in an area considered typical of the tropical American hillsides. Decision-makers needed accurate and up-to-date information. DEMs provide aspect, altitude, and slope data that can be used as input to agricultural and hydrological simulation models which are used to understand the consequences of proposed changes to the agroecosystem of the area. In rugged landscape, such as that encountered in hillsides, height and aspect change dramatically.
over short distances, and errors can accrue rapidly in the absence of dense (and expensive) datasets. Thus, DLMs must be checked for accuracy before being included in any decision support system.

The site chosen covers 30 km² within the Rio Ovejas watershed in the Andean foothills of southern Colombia (Figure 1). It was considered suitable because of its physical characteristics, manageability, accessibility, and the availability of data. The region has an elevation range of 1400 to 2200 m and can be considered typical of many hillside areas found in the Andean regions of South America. Two large rivers with complex networks of tributaries dissect it. There are many v-shaped valleys with steep lower slopes and high plain areas. Thus the site was considered to have a sufficient range of topography on which to test the representational capabilities of DLMs for hillsides.

Both cartographic and photographic data were available for the area at a range of scales. CIAT was involved in a watershed study in the area and thus had already recorded several photo-control points and had set up a permanent GPS control station nearby. The area had to be of manageable size in terms of time spent in the field, computer model size (number and diversity of pixels for raster storage), complexity and processing time. It was considered that 30 km² was large enough to include a diverse range of landscape features yet small enough to be handled in an efficient manner. The area is easily accessible by car and the road network within the area is good. However, because of steep slopes in the area and the roads being largely unpaved, many become impassable after heavy rainfall. Thus, all fieldwork had to be conducted in the dry season.
2.2 Data collection and preparation

Five topographic maps from 1:10,000 to 1:200,000 were used as cartographic input to create the test DLMs. The Instituto Geografico Augustin Codazzi (IGAC) produced these maps. They are horizontally referenced using a national Transverse Mercator Projection with the central meridian positioned through Bogota, the capital of Colombia. They are vertically referenced using Mean Sea Level in Buenaventura, a city located on the Pacific coast, 150 km north-west of the study area. The contour lines were derived from aerial photographs using acro-triangulated photo-identifiable points collected by IGAC. Colombian maps have no documented accuracy specifications. The contour and river arcs were digitized for each model for the 30-km² study area and also for a 500-m zone of interpolation surrounding its boundary.

Names were assigned to the DLMs according to the scale of the input data source and the vertical interval between contours digitized to produce them. For example, Model 10-25 was produced from a topographic map with a source scale of 1:10,000 with a vertical contour interval of 25 m and Model 25-100 from a topographic map with a source scale of 1:25,000 with a vertical contour interval of 100 m. Figure 2 shows the time taken to prepare each of the models. The tasks involved in preparing and correcting a DEM can be divided into seven categories:

1. Calculating values refers to the process of manually assessing and marking altitude values on the maps to ease the digitising process.

2. Digitising includes the setting up of tie points and digitising of all contour lines, rivers and spot heights.
For revision and correction the digitised maps are then compared to the originals to
detect errors such as contour misplacement or missing arcs.

When entering and revising attributes altitude information is added and errors sought.
The time taken for this process depends mainly on contour interval rather than on the scale of the map.

For map joining edges are matched where more than one map sheet is used.

In pre-processing for Hutchinson's the data are then prepared for input to
Hutchinson's interpolation algorithm (see under 2.3). This involves ensuring that all
streams are pointing downstream and that a known sinkhole coverage is prepared
where necessary. The latter is a coverage that contains information about natural
depressions or waterholes in the landscape and is used to ensure their preservation in
the final DEM. All other depressions were treated as data errors and removed. Other
pre-processing steps require preparing an interpolation boundary and generalising
contour lines to minimise data concentration along contours. The time taken for this
stage increases as map scales increase.

The final stage is error correction. Some errors only become apparent after an initial
DEM is produced and checked for accuracy. Contour comparisons and slope profiles
can reveal rivers that do not follow valley floors incorrectly coded contours and
missing lakes and areas needing supplemental altitude data. More errors of this kind
tend to occur in coverages that have been digitised from smaller-scale data because of
the close proximity of contours on the map and the data complexity. Thus map size
and time spent on this phase are inversely related, that is, the smaller the scale the more post-processing work is needed to maximise return from the data.

The costs displayed in Figure 3 provide a useful indicator of the worth of each model when examining level of accuracy in later sections. In calculating costs it was assumed that a trained digitising operator could perform tasks 1-5 whereas tasks 6 and 7 would require someone with a higher skill level, including a good knowledge of DEM production methods and interpolation algorithms. Thus, costs were calculated according to wage levels, assigning US$8 per hour for tasks 1-5 and US$15 per hour for tasks 6 and 7.

The differences in cost appear minimal with regard to models 100-50, 25-100, 100-100 and 200-100. However, the size of the study area is relatively small (30 km²) and minor differences at this level could lead to significant savings when working with larger areas. The estimated costs per square kilometre are shown below. A degree of caution should be exercised if using this to calculate DEM production costs for larger areas.

Cost per model per square kilometre (US dollars)

<table>
<thead>
<tr>
<th>Models</th>
<th>10-25</th>
<th>10-50</th>
<th>25-50</th>
<th>100-50</th>
<th>10-100</th>
<th>25-100</th>
<th>100-100</th>
<th>200-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>33.67</td>
<td>24.87</td>
<td>19.80</td>
<td>18.43</td>
<td>20.57</td>
<td>18.17</td>
<td>17.23</td>
<td>17.10</td>
</tr>
</tbody>
</table>

Considerations such as the number of map sheets that the area covers should be taken into account because of the additional tasks involved in "stitching" multiple map sheets together.
DEMIs that include large tracts of flatter terrain would cost much less because of the reduction in contour lines and the increase in resolution required to represent them (see under 2.3). It can also reasonably be assumed that as the size of the area increases the cost-per-square-kilometre will decrease. In such cases large amounts of data can be handled simultaneously saving processing, checking and verification time per unit area of land.

Additional costs have not been included here (e.g., hardware, software, running costs, map acquisition, and training) because they may vary greatly from one organisation to another. The figures shown above should be considered to act as a good indicator of the relative cost of producing each DEM, all other factors being equal.

Although these costs may appear high, they should be considered in context with the worth of the information. Field survey costs would be much higher if an attempt was made to collect altitude, slope and aspect data in a regular grid pattern over such a large area. Much time and expense would also be incurred in translating field survey results to digital form for analysis. DEM production does have high initial costs, but once produced the model is easy and cheap to manipulate and can be integrated into a wide range of agricultural applications.

2.3 DEM production

The procedure for generating a DEM from topographic maps can be divided into three stages. First, a data structure is chosen with which to represent the surface and, in the case of raster representation, an appropriate cell size selected. Next, a method of interpolation is selected to transform the source data to a continuous elevation surface. Finally, some standard error
checking procedures should be undertaken to ensure that the DEM is an accurate reflection of the data used to produce it.

A choice can be made from different data structures to store the elevation data. The most commonly used are the Triangulated Irregular Network whereby contiguous, planar triangles are fitted to the input data points (Peucker et al. 1978) and the Grid (or Raster-based) data structure, which uses a matrix to store elevation values at regular intervals. A Grid structure was chosen for this study for several reasons. First and most important, its format is easy to manipulate to derive secondary characteristics from the surface (e.g., slope or aspect) and the size of the data blocks can be standardised for multiple-grid comparisons. Well-suited to overlay and other spatial analysis procedures, the Grid structure can be linked to the many other coverages in CIAT that are also stored in this format. Finally, the data are stored in a compact structure using run length encoding, a compression technique that stores long sequences of numbers as single references. Eleven models were produced at a resolution of 5 m and later aggregated to larger cell sizes to investigate the relationship between cell size and slope values. The technique makes it possible to open and work with a many grids simultaneously.

When using a Grid structure the most important consideration is the size of its pixels. This is known as the grid cell resolution. The resolution chosen will directly affect the level of generalisation introduced to the data as it determines the size of each block of land that a single value represents in the grid. This is particularly important when determining slope values because it affects the surface area that contributes to each slope value calculation. Although as small a cell size as possible is desirable to “fit” the terrain closely, the size chosen must also
enable efficient handling and storage of data and attempt to minimise data redundancy. A resolution of 5 m was chosen as a base resolution for each of the models for the following reasons.

1. The complexity of the landscape or the level of detail present in the landscape.
2. The level of detail available in the source data. Important detail may be lost if a cell resolution is overlapping two or more contour lines in steep areas. Retaining as much of the input data as possible is desirable without generalising it and losing valuable information.
3. The project application. The nature of the application is also significant in selecting a cell resolution. With a large-scale project where only a general idea of the regions with high altitude and slope are required, more can be gained by generalising the surface (processing times will be faster and production costs lower) than by representing it with fine detail. In the CIAT Hillsides Program, agricultural modelling projects are typically concerned with small-scale farming plots where many of the farms have less than 5 ha of land.

All models were produced at a resolution of 5 m and later aggregated to larger cell sizes to investigate the relationship between cell size and slope values.

To produce a DEM, a method of interpolation must be developed to transform a set of
discrete altitude data points (i.e., contour data and spot heights) to a continuous dataset (i.e., a DCM) Numerous algorithms exist for this purpose (e.g., Kriging, Inverse Distance Weighting, Linear Regression Analysis and Splines) and a choice between them should be based on the nature of the input data, the type of landscape and the data structure chosen to store the DEM. For the present purpose, Hutchinson's (1989) method of interpolation was chosen to generate the DEMs, based on its proven performance in a series of accuracy tests conducted in similar terrain in October 1993 (Rimcon, 1995). Gao (1997) states that Hutchinson's method of gridding if used for interpolation may achieve a higher level of accuracy. The method uses a grid-based technique of iterative finite-differences that honours input altitude data points and drainage channels according to a user-defined set of "accept or reject" tolerances. A successively smaller grid was placed over the input points and values calculated for the cell centres using the data that fall within each cell. Following each iteration, drainage was assessed through the model and points were eliminated that block flowpaths by less than a specified amount to enable drainage.

Where errors were detected by contour comparison, profiling or histogram analysis, the input data was corrected and Hutchinson's method of interpolation rerun to produce a new model. The process was performed iteratively until the optimal model was produced. This results in a continuous surface that is depressionless and drainable, that is, it contains no walls or sinkholes that would prevent simulated water from flowing to the edges of the DEM. Thus, it is valuable for use in conjunction with agricultural and hydrological datasets. To alleviate any unnatural "terracing" effect, a low-pass 3 x 3 filter was used for smoothing.
2.4 Field control

Two sets of GCPs were needed to produce and quality check the Control DEM. The first set was used to georeference the photographs and to provide vertically referenced data, and the second set to compute the DEM's vertical accuracy. This second set included over 100 points that were selected throughout the area using a random-stratified sample. As this project investigates accuracy, the GCPs had to be accurate to within one half of one pixel resolution of the computed DEM (i.e., 2.5 m). To achieve this level of precision, CLA 1 high-precision GPSs (Leica Systems 2000) were used to provide field control.

Thirty-four random locations were preselected and visited or as close by as possible depending on accessibility. At each of these locations a number of points were collected, situated up to 20 m apart, for 126 checkpoints. Twelve photo-control points were selected from the aerial photographs to use as reference control for the DEM.

Maclaren and Kenne (1989) stated that large-scale stereo photographs are a highly accurate, although expensive, means of producing a three-dimensional model of the terrain. For this study, a model was produced using stereo photographs at a scale of 1:28,000. This was assumed to represent the truest model of the terrain and was used as a control to assess the value of the models produced using topographic maps.

A set of three stereo photographs was selected for the area according to their contrast quality, scale (1:28,000), date (1989), proportion of cloud cover and metadata availability. The negatives were then purchased from IGAC in Bogota, and sent to the USA to be commercially scanned using a high-resolution scanner (25 microns). The GCPs collected in the second
fieldwork were then used to geographically reference the photographs and height values determined using a soft-copy terrain-mapping package (Helava), which IGAC provided. Altitude was not adjusted for forest cover or high-vegetation.

Areas that had been interpolated were identified through photo-analysis and automatically excluded where a height deviation of 15 m or more was apparent between the Control DEM at a 5-m resolution and a parallel model developed at a 10-m resolution. A mask-grid was produced which, as its name suggests, effectively masks out the areas where cloud or uncertainty in height values were apparent leaving 90 reliable GCPs for error checking. The mask-grid was then used to eliminate the same areas from each of the cartographically derived DEMs. A second mask was created to eliminate edge areas from the computed slope values (i.e., where less than nine of the cells in a 3 x 3 window contained data).

In accordance with the accuracy standards specified by the USGS, a Level One DEM (i.e., derived photogrammetrically) should have a vertical root mean square error (RMSE) of not more than 7 m. The maximum RMSE permitted is 15 m (USGS 1990). The RMSE is defined as

\[
RMSE = \sqrt{\frac{\sum (z_i - z_t)^2}{n}}
\]  

(1)

where \(z_i\) = interpolated DEM elevation of a test point, \(z_t\) = true elevation of a test point and \(n\) = number of test points. In this study the computed RMSE was 4.26 m---well within the USGS error specifications for a Level One DEM. Of the checkpoints tested, 83% were within 5 m of the DEM-derived z-value, which is a good fit considering that the DEM cell resolution was 5 m. Vertical errors of up to 4 or 5 m per 5-m ground interval can be anticipated in a complex.
landscape such as this. In effect, 10% of the points showed an error greater than 50 m and 15% showed more than 30 m. Maximum error was 244 m. Note that transformation of the elevation for WGS84 and of IGAC maps has been completed based as a fit between GPS values (WGS89) and map value. The WGS84 surface was consistently higher than IGAC with a mean difference of 26.5 m. Each of the DEMs was assessed for accuracy based on their degree of similarity to the Control Model.

3 Results and discussion

The following analysis and conclusions rely on the assumption that the Control Model represents "truth" in the field. Several factors should be kept in mind regarding the results of this analysis.

- The presence of noise in the central model
- Bias in GCPs' selection
- IGAC maps' accuracy standards
- Results are specific to this type of landscape

3.1 Altitude

The models were first assessed with regard to the accuracy of their altitude values. Table 1 shows the descriptive statistics for each of the DEMs. After removing cloud from each of the models 991,573 cells (24.79 km²) were evaluated. At first glance the figures in Table 1 reveal a
high degree of similarity between the models. Note that the range of altitude values narrows for models 1-9 as the precision of the source data decreases. This is because the altitude values were determined using an averaging technique which, together with the lack of input "peak" or "pit" data in the small-scale models, produces a smoothing effect resulting in "higher lows" and "lower highs" than those that exist on the "true" altitude model. A test of correlation was then performed on each pair of models to provide an initial indication of the relationships between "true" and calculated altitude values.

All populations were normalised using z-scores. Then the correlation coefficient ($r$) was calculated for each pair of models using the total populations (i.e., over 900,000 values). Thus all of the $r$-values can be considered significant at the 100% probability level. The correlation matrix in Table 2 shows results. It clearly shows a strong positive relationship between each of the models and the "true" surface, with all values for $r$ lying close to 1.0. Some preliminary observations can be drawn from this data with respect to the relative strengths of the models. First, the correlation coefficients in column one for model 10-25 (0.9977) and model 10-50 (0.9972) are similar. The value for 10-100 is only slightly lower (0.9933). All these models were based on the same map scale (1:10,000) but a different number of contour lines digitised in each case. The closeness of these figures suggests that it may be more cost-efficient to digitise every second or indeed every fourth contour on a 1:10,000 map rather than every single contour. Second, the coefficients for the degree of correlation between the Control as compared with model 25-100 (0.9886) and model 100-50 (0.9866) imply that the contour interval itself is not the
only determining factor. In this case, the closer contour interval of 50 m produced a model that was actually less similar to "true" altitude than digitising with a contour interval of 100 m because of the difference in base map scale that was used. Thus, a larger scale with less contour lines may be preferable to a smaller scale map with more contours. This suggests that the contour interval and the base map scale should be considered together before selecting the optimal data source for a DEM.

To examine the absolute differences that exist between the models, the altitude values were then compared to "true" altitude on a cell-by-cell basis. The differences were reclassed into error categories and results charted (Figure 4). For this study, a height difference of up to 7 m was considered "good", bearing in mind the surface area of each cell (25 m²) and the probability for height alteration within each individual window. A difference of up to 15 m in height allows a two-cell horizontal placement error, which is considered realistic given the scales of the input data sources. Although not desirable, this was considered acceptable for the type of modelling undertaken. Thus, the percentage of each model that falls within classes 1 and 2 is termed "usable" and is hereafter regarded as an indicator of the value of each of the models.

Note that in model 10-25 (Figure 4), 91% of the cells are considered usable, as opposed to only 50% in model 200-100 at the other end of the scale. However, these figures should be considered with regard to the time taken to produce each of the models (shown in italics). Models 100-100 and 200-100 show poor results, but considered with regard to the short time taken to produce them (almost one third of the time and consequent expense of model 10-25), and then they may be a reasonable option for producing a general representation of the terrain. Model
100-50 has only 54% of usable terrain, which is not much better than these models and actually took longer to produce than model 25-100, which shows better results (60%)

If a cut-off level of the usability of a model is set at 68% (i.e., over 68% of the cells must be within 15 m of "true" altitude otherwise we cannot use the model), only four models are worthy of further discussion 10-25, 10-50, 25-50 and 10-100. Significantly, they include all three models that relied on the 1 10,000 scale map as a base data source. This implies that even if a contour interval is closer on a smaller-scale map, the level of detail that is lost regarding line positioning will have knock-on effects in the process of producing a DEM.

Of the four models, the differences that occur between models 10-25 and 10-50 are so minimal that the 10-50 model is preferable to the 10-25 in terms of time and costs saved in the production process (it takes less than 75% of the time taken to produce model 10-25 and is US$250 cheaper). Models 10-100 and 25-50 also show similar levels of accuracy, with 71% and 77% of each (respectively) falling within an acceptable level of accuracy. As they take about the same time to produce and model 10-100 costs only US$127 more per km², a decision between them may depend on other factors such as their ability to represent slope accurately (see under 3.2). Another factor affecting the decision may be how long is spent on each individual phase of the production process. For example, the total times may be similar but Figure 2 clearly shows that the amount of time spent in the digitising laboratory (tasks 1-4) is lower for model 10-100. This means that a higher proportion of the processing time for this model is spent on tasks that require a higher skill level. It may be desirable to distribute the labour such that the more highly skilled personnel will have more time for other work.
Clearly an informed decision should be based on an understanding of the time and money available, together with the level of accuracy required. Model 10-50 appears to provide a good compromise at the top end of the field and model 25-50 could be used to produce a less accurate, although usable model at three-quarters of the cost. The models produced using 1 100,000 and 1 200,000 source scale data have relatively low standards of accuracy and their use should be avoided if intending to conduct research at a local level (as in this case).

3.2 Slope

A number of algorithms can be used for deriving slope values from a DCM. Most of these operate by fitting a function to four or more of the eight elevation values that surround the central cell (Evans 1980). A 3 x 3 analysis window or "kernel" is passed over the grid and slope values are assigned to each cell based on the relationship between its height value and the heights of its immediate neighbours. Calculation methods include determining the slope of the steepest fall or rise (Goetz, cited in Theobald 1992), calculating directional finite differences within the window, with and without weighted kernels, (Sharpiack and Akin 1969 Horn 1981) and using multiple linear regression to fit a surface to the data points (Skidmore 1989). The latter assesses Horn's method for accuracy (selected for this study) and compares it favourably with several other common methods for calculating slope.
\[3 \times 3 \text{ filter}\]

\[
p_W = \frac{[(Z_{++} + 2Z_{+0} + Z_{++}) - (Z_{-+} + 2Z_{00} + Z_{--})]}{8 \phi_x}
\]

\[
q_W = \frac{[(Z_{++} + 2Z_{0+} + Z_{++}) - (Z_{+-} + 2Z_{0-} + Z_{--})]}{8 \phi_y}
\]

Slope \( x \) for \( Z_{00} \Rightarrow \tan x = \sqrt{\left(p_w^2 + q_w^2\right)}\)

where \( 8 \phi_x \) is the grid cell resolution from west to east and \( 8 \phi_y \) from south to north.

\[\text{Table 3 shows the descriptive statistics for each of the slope models.} \]

Although the range of values increases as the accuracy of the input data decreases, it should be noted that the mean slope value is lower. As the standard deviation remains relatively stable, this suggests that higher incidences of lower slope values occur in the models with lower data density. That is, it appears as if the slope values are being underestimated when represented by models produced from smaller-scale data. To examine this observation in more detail, nine samples of 475 cells each were taken from the normalised populations of the models. First, Analysis of Variance (the \( F \)-ratio test) was used as described in Ebdon (1981) to establish if a significant difference existed between the populations. The following hypotheses were set:
$H_0$: Samples were taken from identical populations

$H_1$: At least one sample was taken from a population with a significantly different distribution to the other samples

An $F$-value of 18.09 was computed and the Null Hypothesis rejected at the 0.0001 probability level. The Tukey-Kramer Studentised Range (Honestly Significant Difference) Test was then used to examine the character and extent of the differences that existed between each of the populations (Kramer 1956). This is a multiple comparison means test similar to the $t$-test but the probability of conclusive errors is lower as the Maximum Experimentwise Error Rate (or probability that the null hypothesis will be falsely rejected) is controlled, assuming equal sample sizes (SAS Institute Inc 1988). A pair of sample means are considered to be significantly different if

$$\left( \mu_i - \mu_j \right) / s \sqrt{v_i + 1/n_j} / 2 \cdot q(\alpha, k, v)$$

where $q(\alpha, k, v)$ is the $\alpha$-level critical value of a studentised range distribution of $k$ independent normal random variables with $v$ degrees of freedom.

Three "Tukey" groupings were identified and are listed in Table 4. Using the identified groupings it can be said that the mean slope values computed by models 25-100, 100-100 and 200-100 differ significantly from the "true" slope values and are underestimating by an average of
The slope values calculated by Models 10-50, 25-50, 100-50, and 10-200 are between 2° and 4° less than the "true" slope values; the latter two models tending towards the larger errors. Model 10-25 is the only one that exhibits no significant mean deviation from the "true" slope values.

These findings confirm the assumption that models derived from small-scale map data (i.e., 1:100,000 or 1:200,000) tend to underestimate slope values. It can be explained by the lack of topographic detail available in the source data, which leads to larger tracts of interpolated land values. Because of the nature of most interpolation algorithms used, these values will be placed along an unrealistically smooth scale between known data points. Although these findings are specific to the methods of interpolation and type of landscape that were modelled for this study, clearly a general correction factor should be applied to models derived from small-scale data. The correction factors below can be used as a "rule-of-thumb" compensation for generalised slopes in studies where only small-scale maps are available.

Correction factors (CF) for underestimated slope values (Cell resolution = 5 m)

<table>
<thead>
<tr>
<th>Model</th>
<th>10-25</th>
<th>10-50</th>
<th>25-50</th>
<th>100-50</th>
<th>10-100</th>
<th>25-100</th>
<th>100-100</th>
<th>200-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (degrees)</td>
<td>0</td>
<td>+2</td>
<td>+2</td>
<td>+3</td>
<td>+4</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
</tr>
</tbody>
</table>

Correlation coefficients were then calculated for each of the slope grids as compared to the Control Model using the methodology previously described. The low correlation values initially calculated (Table 5) reflect the inherent structural differences between the Control DEM and the models derived through interpolation.

To decrease the roughness of the terrain visible in the Control Model and alleviate minor
errors in elevation values the surface was smoothed using a 3 x 3 low-pass filter. The correlation coefficients after this initial smoothing of the data show some improvement (Table 5 column 3) but these figures are still too low to produce a trustworthy estimate of slope values. The models could be smoothed repeatedly to iron out the surface undulations and thus increase the correlation between "true" and calculated slope. The resulting slopes would be more similar but highly generalised and would represent a surface area far larger than twice the cell size because of the averaging of neighbouring cells to achieve the smoothed effect. Thus it was considered undesirable to smooth the data further and alternative explanations were examined for the high internal deviation between the DEMs.

3.2.2 Potential sources of error

A thorough analysis of potential sources of errors in both the reference DEM and landscape factors showed that most errors were caused by structural differences within the models. The high resolution of the DEMs is the most influential factor affecting the level of detail (i.e., surface undulations) in the models. Although minimising the cell size to maintain terrain complexity for altitude values is desirable, the local errors that result from this level of detail contribute heavily to the deviation of slope values. The only way to increase slope accuracy in this case is to increase the size of the area that the slopes represent or select a coarser resolution by averaging thus avoiding local error and producing more generalised slope values. The relationship between cell resolution and slope correlation was therefore examined to determine whether an optimal cell resolution that would reflect the quality of the input data could be recommended for each of the
models

Figure 5 illustrates the effect that increasing the cell size has on the correlation of slope values. It clearly shows that as the cell size is increased the degree of correlation increases sharply up to a sill point after which the rate of change slows tending towards 1.0. It seems safe to assert that this sill point is the lowest the cell size can go to produce a reasonable definition of slope. Below this level unwanted noise is present in the data. This graph clearly shows that a different cell size suits a different level of input data. Note that the models with a wider contour interval have consistently lower correlation with "true" slope despite a higher degree of correlation to "true" altitude (Figure 6). This confirms the importance of a narrow contour interval to maintain accurate intra-cell relationships. Therefore it is important to determine the cell size best suited to the level of detail available in the source data.

To determine such cell size the correlation coefficients were first compared with the internal cell deviations to determine a means of defining a level at which the $r$-value represents a usable model. The level of $6^\circ$ was chosen as the cut-off for the accurate representation of slope. Beyond this level errors were considered too high for use in agricultural databases. Figure 6 shows the relationship between the correlation value and the proportion of the model that falls within $6^\circ$ of "true" slope. Thus the regression line can be used to determine the quality of a model at any resolution with regard to the proportion considered usable. If a cut-off point of 68% (i.e., over 68% of the model must be within $6^\circ$ of "true" slope) is set, a model will only be accepted with a correlation coefficient of 0.66 or more (i.e., $68 = \left[0.66 \times [0.66] + 11.228 \right]$).

Using this method and referring to Table 4 it can be determined at what resolution each
model could be considered "usable" for slope. Thus, in this case (with a cut-off of $r = 0.66$) it can be stated that models 10-25 and 10-50 are best represented with a cell size of 20 m, model 25-50 at 40 m, and models 10-100 and 25-100 at 70 m. Models 100-50 100-100 and 100-200 are not considered usable with a cell size of less than 120 m. Using Table 4 with Figure 6 a cell size and model can be selected that suit the level of accuracy required. However, it should be stressed again that increasing the resolution results in larger and larger slope surfaces being represented.

Before using any of the derived slope grids, whether these generalised slopes are good enough for the application must be decided. In the case of a Hillsides Research Project, a cell size of 30 m is probably the maximum level of precision at which to work. In this case, only models 10-25, 10-50 and 25-50 are considered usable. The other models may provide a good general representation and a relative indicator of high or low slope areas but cannot be relied upon for large-scale modelling purposes.

4 Conclusions

In producing a DEM from topographic maps, enlarging the interval between digitised contours can save time and expense. This research indicates that it would be cost-effective to digitise every $n$th contour on a map rather than every single line provided the new interval is less than 25 m wider than the original interval when modelling altitude or slope. This interval can be increased to 50 m wider than the original contour interval in cases where modelling altitude only is required. With regard to slope determination, the contour interval has more influence than the map scale in providing realistic slope representations. Models with a wide contour interval were less accurate in modelling slope at acceptable resolutions (under 40 m), regardless of the scale of
the source map

Cartographic data sources at a scale of 1:100,000 or 1:200,000 with a contour interval of 100 m or more provide insufficient detail to accurately represent slope in the hillside areas. But if these are the only source available in a designated research area, it may be possible to estimate "true" slopes by applying a correction factor to the calculated slope values. Small-scale maps such as these have been proven to consistently underestimate slope by an average of 5°. However, even if a correction factor is applied, high incidences of error will probably occur and this method is recommended only as a last resort.

The optimal cell resolution of a DEM is not necessarily "the smaller the better" if intending to derive slope values in a complex landscape. A resolution that is too small incorporates too much noise (i.e., local deviations) in the data and can lead to erroneous slope calculations. In selecting a cell size to model slope, both the type of landscape and the level of detail in the source data must be taken into account. With a base map scale of 1:100,000, the best resolution is upwards of 70 m despite the desirability of modeling this kind of landscape at less. If the base map scale is unable to represent slope accurately at the desired resolution (i.e., up to 30 m) then it should be considered unsuitable as an input data source for the DEM.

Although low-pass filters can be used to "iron out" local errors in the surface, they should not be used too liberally as such methods are ultimately affecting the slope values in the same way as increasing the cell size. Namely, slopes derived from multi-filtered data will be representing a surface area that is several times larger than the cell itself.

With regard to using DEMs derived from topographic maps as a source for geo-correcting
remotely sensed images only scales of 1 10 000 and 1 25 000 (less reliable) would apparently be usable. As the slopes derived using Horns' algorithm represent up to twice the cell size a resolution of 15 m would be considered desirable to provide slope input to a 30-m resolution satellite image. Unless using a different algorithm, stereophotos or other methods should be considered as a more suitable data source from which to derive the DEM.

Finally and most importantly, this information goes some way towards providing users with an awareness of the level of accuracy that they can expect of a DEM. Clearly the scale of the source data from which it has been produced is fundamental to the dependability of the model. Care should be taken to obtain specifications regarding production methods and error checking procedures and their results before attempting to incorporate a DEM or any of its derived products into a spatial modelling system.

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Table 1 Descriptive statistics for altitude models

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<thead>
<tr>
<th>Model</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Standard</th>
<th>Range</th>
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Table 2 Correlation matrix for altitude models

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Table 3  Descriptive statistics for slope models  Slope values represent a 10 m² surface area

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<th>Maximum</th>
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<th>Standard</th>
<th>Range</th>
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<tr>
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<td>17.7</td>
<td>10.5</td>
<td>71.1</td>
</tr>
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<td>10.8</td>
<td>80.6</td>
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<tr>
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<td>10.7</td>
<td>82.9</td>
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<tr>
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<td>71.3</td>
</tr>
<tr>
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<td>72.9</td>
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<td>14.9</td>
<td>10.7</td>
<td>82.7</td>
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</table>

1  minimum slope value is 0 for all models
Table 4  Tukeys Studentised (Honesty Significant Difference) Range Test for Dignal Elevation
Model slope populations

<table>
<thead>
<tr>
<th>Model</th>
<th>Tukey grouping</th>
<th>$\sqrt{\mu}$</th>
<th>Control $\mu$</th>
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<td>10 25</td>
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<td>AB</td>
<td>3.96079</td>
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<td>25 50</td>
<td>AB</td>
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<td>100 100</td>
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<td>200 100</td>
<td>C</td>
<td>3.54269</td>
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</table>

1  Critical value of Studentised Range = 4.389 minimum SD = 0.2713 0.05 df 4266 MSE

2  Models marked with the same letter are not considered significantly different
Table 5  Correlation coefficients for slope models as compared to true slope

<table>
<thead>
<tr>
<th>Model</th>
<th>Control model</th>
<th>Improvement in r</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient correlations r</td>
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</tr>
<tr>
<td></td>
<td>Raw data</td>
<td>After smoothing</td>
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<td>0.39581</td>
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<tr>
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<td>0.34262</td>
<td>0.39042</td>
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<td>0.26454</td>
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<tr>
<td>9 200 100</td>
<td>0.24281</td>
<td>0.27580</td>
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</table>
(missing-need a map of 30 km² within Río Ovejas watershed in the Andean foothills of southern Colombia)

Figure 1     Location of the study area
Figure 2  Digital Elevation Model (DEM) preparation times
Figure 3  Cost in US dollars for producing each Digital Elevation Model
Figure 4  Difference between Digital Elevation Model altitude values and true altitude (all 991,573 cells have been classified)
Figure 5  Effect of increasing cell size on correlation of slope values
Figure 6  Cut-off point for determining model usability